1.1 (5)

$x \to \infty : f(x) = \frac{2e^{2x} - 10}{e^{2x} + 3} \to \frac{"\infty"}{\infty} \overset{L.H}{\rightsquigarrow} \frac{4e^{2x}}{2e^{2x}} \to 2 \;;\; y(x) = 2$

$x \to -\infty : f(x) \to \frac{-10}{3} \Rightarrow y_2(x) = -\frac{10}{3}$

1.2 (5)

$f(x) = 0 \Rightarrow 2e^{2x} - 10 = 0 \Leftrightarrow e^{2x} = 5 \Leftrightarrow x = \frac{1}{2}\ln(5) = \ln(\sqrt{5})$

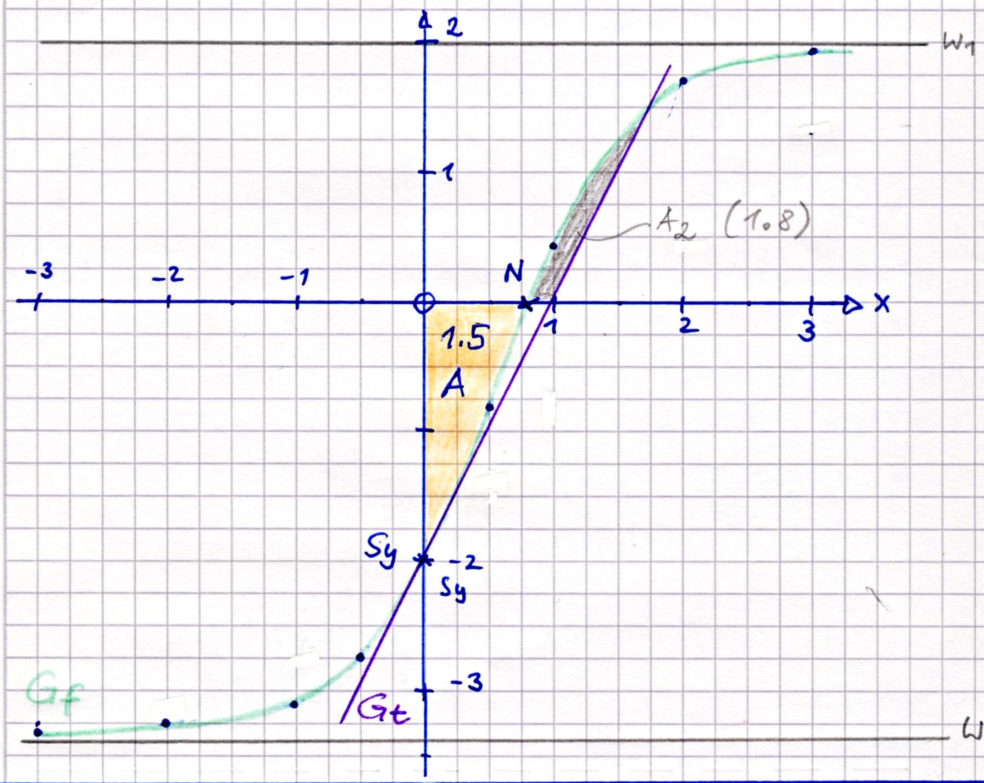
$S_y(0|f(0)) = S_y(0|-2)$ $(\ln\sqrt{5} \approx 0{,}80)$ $N(\ln(\sqrt{5})|0)$

$f'(x) = \frac{(e^{2x}+3)\cdot 2e^{2x}\cdot 2 - (2e^{2x}-10)\cdot e^{2x}\cdot 2}{(e^{2x}+3)^2} =$

$= \frac{4e^{2x}(e^{2x}+3-e^{2x}+5)}{(e^{2x}+3)^2} = \frac{8\cdot 4e^{2x}}{(e^{2x}+3)^2} = \frac{32e^{2x}}{(e^{2x}+3)^2}$

$32e^{2x} > 0$; keine NST $\Rightarrow$ kein Extr. ; $f'(x) > 0 \Rightarrow$ sms in $\mathbb{R}$

1.3 (4)



1.4 (4)

$F'(x) = \frac{2}{3}\left(\frac{4}{e^{2x}+3}\cdot e^{2x}\cdot 2 - 5\cdot\left(\frac{e^{2x}+3}{e^{2x}+3}\right)\right)$ erw. auf Hauptnenner

$= \frac{2}{3}\;\frac{8e^{2x} - 5e^{2x} - 15}{e^{2x}+3} = \frac{\frac{2}{3}(3e^{2x}-15)}{e^{2x}+3} = f(x)$

Markierung

1.5 (4)

$$A = -\int_0^{\ln(\sqrt{5})} f(x)\,dx = \left[F(x)\right]_{\ln(\sqrt{5})}^{0} = F(0) - F(\ln\sqrt{5}) =$$

$$= \frac{2}{3}\left(4\cdot\ln(e^0+3) - 5\cdot 0\right) - \frac{2}{3}\left(4\cdot\ln(e^{2\cdot\ln\sqrt{5}}+3) - 5\ln(\sqrt{5})\right)$$

$$= \frac{2}{3}\left(4\cdot\ln(4)\right) - \frac{2}{3}\left(4\cdot\ln(8) - 5\cdot\ln(\sqrt{5})\right) = \quad (\approx 0{,}83400)$$

$$= \frac{2}{3}\left(4\cdot\ln(4) - 4\cdot\ln(8) + 5\ln(\sqrt{5})\right) = \frac{8}{3}\ln\left(\frac{1}{2}\right) + \frac{5}{3}\ln(5)$$

1.6 (3)

$$f'(0) = m = \frac{32e^0}{(e^0+3)^2} = \frac{32}{16} = 2\ ;\ S_y(0|2) \Rightarrow t(x) = 2x - 2$$

G_t

1.7 (5)

$$f(x) = t(x) \Leftrightarrow f(x) - t(x) = 0 \quad (= h(x))\ ;\ h'(x) = f'(x) - 2$$

$$h(2) = f(2) - t(2) = \frac{2e^4 - 10}{e^4 + 3} - 2\cdot 2 + 2 \approx -0{,}2777867\ldots$$

$$h'(2) = f'(2) - t'(2) = \frac{32e^4}{(e^4+3)^2} + 2 \approx -1{,}473363645\ldots$$

$$u_1 = 2 - \frac{-0{,}2777866996}{-1{,}473363645} = 1{,}811(46087)$$

1.8 (3)

$$A_2 = \int_{\frac{1}{2}\ln(5)}^{4} f(x)\,dx - \int_1^4 t(x)\,dx\ ;\ \text{Markierung}$$

2.1 (5)

$\begin{pmatrix}1\\0\\-2\end{pmatrix} \neq k \cdot \begin{pmatrix}1\\1\\0\end{pmatrix} \Rightarrow$ g und h_a nie parallel

$$\begin{aligned} 3 + \lambda &= a + \mu \\ -5 &= -2a + \mu \\ 2 - 2\lambda &= 6 \iff \lambda = -2 \end{aligned}$$

$$\begin{aligned} 1 &= a + \mu \\ -5 &= -2a + \mu \end{aligned} \quad \rightarrow -6 = -3a \iff a = 2 \quad \rightarrow \mu = -1; \quad -5 = -4 - 1 \text{ (w)}$$

Für $\underline{a = 2}$ (und $\mu = -1$; $\lambda = -2$) schneiden sich g u. h_2, sonst sind sie windsch.

2.2 (3)

$$\vec{n}_E = \begin{pmatrix}1\\0\\-2\end{pmatrix} \times \begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}0+2\\-2+0\\1-0\end{pmatrix} = \begin{pmatrix}2\\-2\\1\end{pmatrix}; \quad E: \begin{pmatrix}2\\-2\\1\end{pmatrix} \circ \begin{pmatrix}x_1-3\\x_2+5\\x_3-2\end{pmatrix} = 0$$

$$\Rightarrow E: 2x_1 - 2x_2 + x_3 - 6 - 10 - 2 = 0 \iff \underline{E: 2x_1 - 2x_2 + x_3 - 18 = 0}$$

2.3 (5)

$$l: \vec{x} = \tau \begin{pmatrix}2\\-2\\1\end{pmatrix} \text{ in } E: 2 \cdot 2\tau - 2 \cdot (-2\tau) + \tau - 18 = 0 \iff \tau = 2$$

in l: $L(2 \cdot 2 | -2 \cdot 2 | 1 \cdot 1) = L(4|-4|2)$

$$d = |\vec{OL}| = \sqrt{16 + 16 + 4} = \sqrt{36} \Rightarrow \underline{d = 6}$$

2.4 (4)

x_3-Achse: $a_3: \vec{x} = \sigma \begin{pmatrix}0\\0\\1\end{pmatrix}$ in $E: \sigma - 18 = 0$ in a_3: $\underline{S(0|0|18)}$

$$\sin(\alpha) = \left| \frac{\begin{pmatrix}2\\-2\\1\end{pmatrix} \circ \begin{pmatrix}0\\0\\1\end{pmatrix}}{\sqrt{4+4+1} \cdot 1} \right| = \frac{1}{3}$$

$$\alpha = \sin^{-1}\left(\frac{1}{3}\right) \approx \underline{19{,}471^\circ}$$

2.5 (3)

Sym. Punkt $O^* = 2\tau$ in l (vgl. 2.3): $O^*(8|-8|4)$

$$s: \vec{x} = \vec{S} + \alpha \, \vec{O^*S} = \begin{pmatrix}0\\0\\18\end{pmatrix} + \alpha \begin{pmatrix}0-8\\0+8\\18-4\end{pmatrix} = \begin{pmatrix}0\\0\\18\end{pmatrix} + \alpha \begin{pmatrix}-8\\8\\14\end{pmatrix}$$

oder: $\underline{s: \vec{x} = \begin{pmatrix}0\\0\\18\end{pmatrix} + \alpha' \begin{pmatrix}-4\\4\\7\end{pmatrix}}$